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THE ROLE OF THE FIRST AND SECOND MODES IN COMPRESSIBLE BOUNDARY-LAYER TRANSITION

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At present there is no complete theory that can predict the transition location in a compressible boundary layer. In practice, however, the well-developed approximate methods based, as a rule, on linear stability theory are used (see, e.g., [1]). In the absence of information on the initial disturbance spectrum in the boundary layer (e.g., in flight tests) it is possible to use the (crude) e^n-method to locate transition. This method has been effective at subsonic speeds in "wind-tunnel" as well as flight tests including three-dimensional boundary layers (see, e.g., [2]). In this method the transition location is fixed when the disturbance amplitude ratio $A = Q/Q_0$ attains the value e^n (Q_0 is the disturbance amplitude at the lower branch of the neutral stability curve, Q is the current value of the amplitude, and n is specified) which is the amplification ratio in the unstable region. The transition Reynolds number determined in such a manner is an integral characteristic of the boundary-layer instability. It can be used to lucidly compare the contributions made by the first and the second modes to the growth of unstable disturbances in the boundary layer and investigate the influence of various factors on both the modes. A comparison of stability characteristics (growth rate, neutral curves, and transition Reynolds number) of the first and the second disturbance modes is the primary objective of the present paper.

1. The basis for this study is the program to compute disturbance amplification rate α_i in the boundary layer with heat transfer [3]. A detailed description of the computational technique to determine the stability characteristics is given in [1, 4].

Consider a compressible, heat conducting, two-dimensional boundary layer (see, e.g., [5] for the system of equations). Computationals are carried out for air flow on an impermeable surface with a specified wall temperature. Almost all computations are carried out for zero pressure gradient flow past a cone. The only exception was the study of the influence of pressure gradient on the amplification ratio.

The system of equations describing the flow in the boundary layer was transformed to a system of ordinary differential equations (for the flow with a pressure gradient local similarity was assumed [5]) which was then numerically integrated (see [1] for details). Numerical integration was used to determine the streamwise velocity and temperature distributions, their derivatives and the variation of viscosity across the boundary layer. These were required for the solution of the stability equations. In order to determine the amplification ratio the Dunn–Lin [6] approximation was used for the system of stability equations with boundary conditions: streamwise and normal velocity as well as temperature

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fluctuations at the wall are zero and are damped at infinity.

The solution to the Dunn–Lin system of equations with the given boundary conditions is described in [1]. New variables are introduced and the Dunn–Lin system of equations (in partial derivatives) is transformed to a system of six, first order, ordinary differential equations. It is numerically integrated using orthogonalization technique. The following assumptions were made in these computations: Prandtl number Pr = 0.72, ratio of specific heats $\gamma = 1.41$, Sutherland viscosity law $\mu = cT^{3/2}/(T + T_S)$ (c is a constant, $T_S = 110.4$°K).

Numerical results from integration were used to obtain information on three-dimensional disturbances in the form of relations $\alpha_i = \Phi(\text{Re}, F, \chi)$, where $F = \omega/\text{Re}$ (ω is the circular frequency), χ is the angle at which the disturbances are propagated (inclination of the wave angle to the streamwise direction), Re is the Reynolds number. The Reynolds number for the flow past a cone is given by $\text{Re} = \sqrt{u_e s/\nu_e 3}$, where u is the streamwise velocity, ν is the kinematic viscosity, s is the coordinate along the body surface, and the index e denotes quantities at the outer edge of the boundary layer. It was assumed that ω is real and $\beta_i/\alpha_i = \beta_r/\alpha_r$ [7], where $\alpha = \alpha_r + i\alpha_i$ is the wave number in the streamwise direction, and $\beta = \beta_r + i\beta_i$ is the wave number in the lateral direction. Then $\chi = \text{arctg}(\beta_r/\alpha_r)$.

The disturbance amplification rate is related to the disturbance amplitude Q by the equation $\text{Real}(d \ln Q/ds) = -\alpha_i^{\text{dimen}}$. Hence it follows that for a cone $\ln|Q/Q_0| = -6\int_{\text{Re}_0}^{\text{Re}} \alpha_i d\text{Re}$ (α_i is the nondimensional amplification rate). This expression determines the ratio of the disturbance amplitude at points with coordinates Re and Re_0 and gives the amplification ratio for the disturbance in the given segment.

Unlike the case of subsonic speeds (where the most unstable disturbances are two-dimensional), it is necessary to consider $\alpha_i = \alpha_i(\text{Re}, F, \chi)$($\chi$ is the wave propagation angle) when M > 1. Here the maximum $-\alpha_i$ at M = 1.5-7 are obtained in the range $\chi \approx 50$-70° for the first mode (for the second mode the two-dimensional disturbances are more unstable). In the present work computations are carried out for the critical angle χ^* which is determined as that angle at which the integral $-6\int_{\text{Re}_0}^{\text{Re}} \alpha_i d\text{Re}$ most rapidly reached the specified value n (n = 9), i.e., when $\chi = \chi^*$ the Reynolds number determined at $A = e^n$ is a minimum (it is tentatively assumed to be the transition Reynolds number Re_{tr}); Re_{tr} being an integral quantity (taking into consideration amplification rates as well as the neutral curve) of the boundary-layer instability, it is considered a fundamental parameter for the comparison of the stability characteristics fo the first (low frequency) and the second (high frequency) disturbance modes. In the case of uniformly distributed disturbance energy spectrum in the boundary layer, the transition location is determined by the mode for which Re_{tr} is less.

It is known that the first disturbance mode is completely analogous to the well-known Tollmien–Schlichting wave from hydrodynamic stability theory. For example, with wall cooling in air flow, the mean velocity, temperature, and density profiles are altered such that the flow becomes more stable relative to the first mode.

The second mode represents another form of acoustic resonance in a shear flow. This type of instability was discovered in [8, 9]. The effect of a number of factors on the amplification ratio of the second mode is discussed in [10]. It was explained that the oblique waves of this type ($\chi \neq 0$) are more stable than two-dimensional disturbances ($\chi = 0$) and surface cooling has a weak destabilizing effect. However, an integral characteristic such as Re_{tr} was not determined for the second mode.

2. Stability characteristics have been computed for M = 1-7°, stagnation temperature $T_0 = 210$-1000°K and temperature ratio $T_w = 0.5$-1.0, and amplification factor at a small velocity (pressure) gradient $\beta = (2\bar{s}/u_e)du_e/d\bar{s} = 0$; 0.01; and 0.1 $\left(\bar{s} = \int_0^s u_e \nu_e \rho_e^2 r_w^2 ds,\right.$ ρ is the density, $r_w(s)$ is the equation of the body contour).

Results of some computations are shown in Figs. 1-5. The variation in Re_{tr} for the first (curve 1) and the second (curve 2) modes at M = 1-7 is shown in Fig. 1 for an insulated cone ($T_w = 1$, $\beta = 0$). The static temperature for all Mach numbers is the same

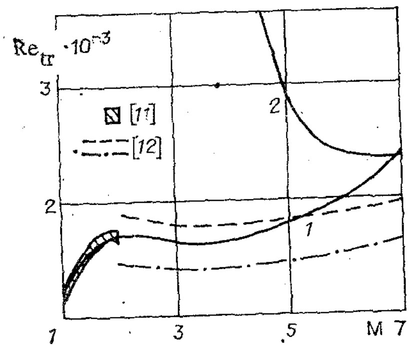

Fig. 1

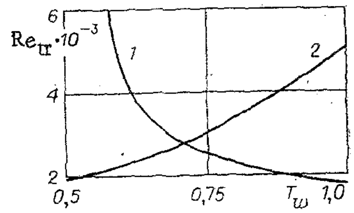

Fig. 2

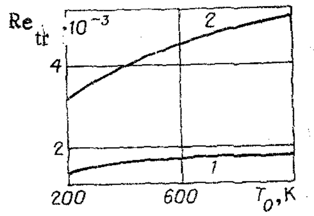

Fig. 5

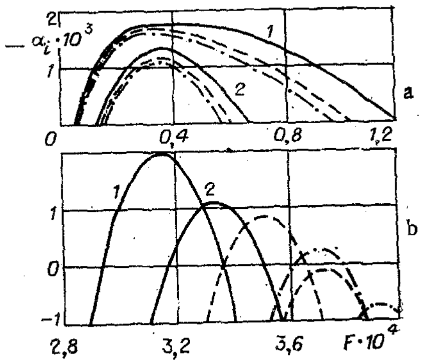

Fig. 3

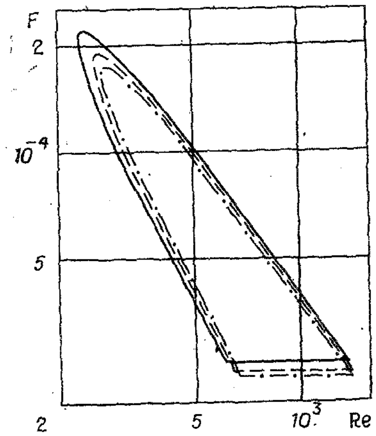

Fig. 4

(T_e = −50°C). A comparison of computed data with flight tests and wind-tunnel tests (flight tests on F-15 (cone, T_w = 1) [11] and ballistic tests [12] (cone, T_w is a variable) is shown in Fig. 1; dashed line corresponds to $Re_1 = (u/\nu)_\infty = 28 \times 10^6$ M^{-1}, dashed-dotted line is for $Re_1 = 11 \times 10^6$ M^{-1}. On the whole (and especially when M = 1-4) the computed and actual (obtained in real situation) Re_{tr} agree very well which indicates that e^n-method ($n \approx 9$) gives a good estimate of transition location. Experiments and computations show that for the first mode there are local maxima (when $M \approx 2$) and minima (when $M \approx 3.5$-4) depending on $Re_{tr} = Re_{tr}(M)$ at $T_w = 1$ and $\beta = 0$ whereas for the second mode under the same conditions there is only a single minimum (when $M \approx 6.5$-7). With an increase in Mach number (starting from $M \approx 7$) Re_{tr} increases for the first as well as the second modes. And approximately when M > 7 for uniformly distributed energy spectrum, the location of transition will be determined not by the first but by the second mode.

It is also necessary to emphasize that the behavior of an integral quantity such as Re_{tr} cannot always be judged on the basis of individual dependences (on Mach number in the present case) of the critical Reynolds number Re_{tr} (minimum Reynolds number at which disturbances start to grow at any particular frequency) and maximum amplification factors. In particular, with a reduction in M in the range M = 1-4, Re_{tr} as well as $(-\alpha_i)_{max}$ increase continuously whereas the relation $Re_{tr} = Re_{tr}(M)$ has a maximum at $M \approx 2$ (which agrees excellently with flight tests [11]). On the other hand, maximum amplification factors for the second mode become larger than those of the first mode when $M \geq 4$, and for the integral characteristics Re_{tr} this occurs when M > 7. For the second mode the maximum of the dependence of $(-\alpha_i)_{max}$ on M takes place at $M \approx 5$, and the minimum of the dependence $Re_{tr}(M)$ occurs in the range M = 6.5-7. All these indicate the need for estimating the transition location based particularly on the integral characteristics of Re_{tr}.

The variation of Re_{tr} for the first (curve 1) and the second (curve 2) modes is shown in Fig. 2 as a function of the temperature ratio (T_w = 0.5-1.0) at M = 4, and T_0 = 937°K (T_w is the ratio of the wall temperature to the stagnation temperature). It is seen that even for $T_w \lesssim 0.7$ in uniformly distributed disturbance spectrum, the transition location is determined not by the first mode but by the second mode; it plays the decisive role for $T_w < 0.55$ for practically any given spectrum. The nature of these variations shown in Figs. 1 and 2 is confirmed by experiments [13] conducted at M = 8 and T_w = 0.93 and 0.48, where it is shown that the transition Reynolds number caused by the second mode decreases with a decrease in T_w.

Figure 3 demonstrates the effect of pressure gradient on the amplification rate α_i of the first (a) and the second (b) modes at M = 4 and Re = 780 (F is the nondimensional frequency). Significant stabilizing influence of negative pressure gradient is observed, especially for the second mode. Since the influence of β on the second mode is more significant than on the first mode, α_i at β = 0 and 0.1 (lines 1 and 2) are shown in Fig.3a and at β = 0 and 0.01 (lines 1 and 2) in Fig. 3b.

The influence of stagnation temperature T_0 on the stability characteristics (amplification rate, neutral stability curve, and conditional transition Reynolds number) is shown in Figs. 3-5 (T_w = 1): solid lines indicate T_0 = 300°K, dashed lines indicate 600°K, dashed - dotted lines denote 900°K. In Fig. 5, line 1 is for the first mode and the line 2 is for the second mode. Figures 3-5 show that an increase in T_0 leads to the stabilization of the first mode (a reduction in $-\alpha_{i max}$ and an increase in Re_{cr} and Re_{tr}; in Fig. 4 (M = 1.5) neutral curves are limited at the lower end by the frequency at which $A = e^9$, and the extreme right point of the curve corresponds to Re_{tr}). The effect of stagnation temperature on the stability characteristics increases with Mach number. The data obtained agree well with results [10, 14] for the influence of T_0 on α_i and Re_{cr} for the first mode. An increase in T_0 also has a stabilizing influence on the second mode at M = 4 (Fig. 3b and 5), which is even stronger than that on the first mode.

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